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Online mathematical competition: using virtual marathon to challenge promising students and to develop their persistence

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ABSTRACT

In what way a virtual summer competition can engage mathematically promising students in a meaningful activity? By stating and investigating this question we looked for an insight into students' perseverance and ability to deal with challenging mathematics beyond the school walls. The participation data were analysed from two perspectives. First, how active were students in problem-solving activities and what the relation between success and persistence is. Second, we looked at the most persistent participants and particular problems that were difficult for them. Our findings can help to improve design of virtual competitions and to attract more students with challenging mathematics.

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CONTEXT AND PROBLEM STATEMENT

The *(Author 1's website name and website address)* has been offering rich mathematical problems on-line for 10 years, which has attracted schoolchildren from all school levels with challenging mathematical problems (Author 1 & Author 2, 2009). In our previous work, we argued that the Problem-of-the-week model used in this project could be suitable for enriching mathematics learning in all participants, particularly the most promising (Author 1, 2009).

Despite *(website's name)* noticeable success, we found that more interactive and diversified forms of activities could be used with these schoolchildren. To this end, we created a new online activity, the Virtual Mathematical Marathon (VMM), on the *(website's name)* website. The project lasted two years during which 80 problems in all were posted, 4 problems twice a week, in the summer. Close to 200 students, during the first year, and more than 100 during the second year submitted their answers to some of these problems on a voluntary basis. Based on available literature, we assumed that participation in the online competition could contribute to the development of mathematically promising students. This would be accomplished through (1) persisting when facing challenges while (2) fostering their abilities of solving non-routine problems over a long period of time.

Renzulli (1978) highlighted three basic traits found in successful outstanding individuals: above average general ability, high level of task commitment, and creativity. A number of studies have explored Krutetskii's characteristics of mathematically able students (Krutetskii, 1976) and Polya's framework of mathematical creativity (Polya, 1973). However, the task commitment seems to be less studied, although Renzulli insisted that "energy brought to bear on a particular problem (task) or specific performance area" is a very important aspect of giftedness (Renzulli, 1978, p. 182). Na et al. (2004) mentioned *task commitment* as an alternative criterion to school marks to identify the mathematically gifted.

The concept of mathematical promise introduced by Sheffield (1999) situates our research in the field of mathematical challenge and giftedness in a much broader sense than it was seen in studies on exceptional abilities and talent. More than 10 years of studying mathematical promise brought attention to tasks that would be rich, interesting, open-ended, and accessible to more students and thus attract the largest possible spectrum of students by finding challenges according to everyone's abilities and needs (Barbeau & Taylor, 2009; Taylor, Gourdeau, & Kenderov, 2004).

In order to build our theoretical frameworks, we conducted a literature review by looking at the role played by competitions, challenging tasks, and virtual environments in the development of mathematically promising and persistent students.

THEORETICAL BACKGROUND

The role of competitions in the education of the gifted

Mathematical competitions in their recent form have more than 100 years of history and traditions. Kahane (1999) claimed that large popular competitions, like the *Australian Mathematics Competition* or the *Kangourou* mathematics competition (Kahane, 1999, p. 139), could reveal hidden aptitudes and talents, and stimulate large numbers of children and young people. A recent study by Robertson (2007, p. 40) of the history and benefits of mathematical competitions reported that success in math competitions and in math achievement in general, seemed to be linked to the love and interest instilled in students in learning and an appreciation for math and problem-solving methods, as well as the opportunity to acquire high-level skills with extra training and third the development of particular culture, which encourages hard work, learning, and achievement. Bicknell (2008) also found many advantages to be gained from the use of competitions in a mathematics program such as student satisfaction, the enhancement of students' self-directed learning skills, sense of autonomy and, co-operative team skills.

The choice of appropriate challenging tasks is also an important condition of success in mathematical competitions to develop students' learning potential. Leikin (2009) claims that such tasks should: be *appropriate to students' abilities*, being neither too easy nor too difficult; *motivate students to persevere* with task completion; and develop mathematical curiosity and interest in the subject. They should also *support and advance students' beliefs* that the creative nature of mathematics, the constructive nature of the learning process, and the dynamic nature of mathematical problems have different solution paths. Finally, they should also support individual learning styles and knowledge construction. The latest explosive development in Internet-based resources and activities brings new dimensions into the models of mathematical challenges (Author 1, 2009).

The role of virtual learning environments

Internet can be a suitable challenging environment for organizing mathematical competitions and problem-solving activities, contributing potentially to the development of mathematical ability and giftedness. The use of technology can be considered as an inclusive form of mathematical enrichment, providing a tool, an inspiration, or a potentially challenging and motivating independent learning environment for any student. For the gifted ones, it is often a means to reach the appropriate depth and breadth of curriculum and advanced learning opportunities, as well as better engagement and task commitment (Author 1 and Author 2, 2009; Johnson 2000; Jones & Simons, 2000; Renninger and Shumar, 2004).

With our VMM, we sought to provide students with an opportunity to discover talent that they do not normally demonstrate in regular classrooms (Taylor, Gourdeau, & Kenderov, 2004). To this end, we considered a marathon as a stimulus for improving students' informal learning. Fomin, Genkin, & Itenberg (2000) described that during the marathon that they conducted on paper-and-pencil basis, their students managed to increase the number of problems they solved, relatively to other their non-competitive settings. Additionally, these students found that face-to-face marathons were more interesting and attractive than other, better known forms of olympiads (idem.). The marathon as a form of mathematical activity over the long period of time could help us to get an insight in the nature of task commitment and its nurturing.

STRUCTURE OF THE VIRTUAL MATHEMATICAL MARATHON

According to our model of the VMM, 20 sets of 4 non-routine challenging problems were posted twice a week on the *(website's name)* website during 10 weeks, from June to August in 2008 and 2009. Every registered member could login, choose a problem, solve it, and submit an answer by selecting it from a multiple-choice menu. The automatic scoring system immediately evaluated students' success producing a score for the problems and adjusting a total score that affected the overall standing.

According to the level of difficulty, scores per problem were determined as follows: level 1(easiest) was scored with 3 points, level 2 with 5 points, level 3 with 7 points, and level 4 (hardest) with 10 points. To support students' participation in the marathon, unsuccessful attempts were nevertheless rewarded with 1, 2, 3, and 4 points respectively. Participants could join the marathon, solve as many problems as they wished, withdraw, and come back at any time. The tasks were developed by a team of experts in mathematics and mathematics education.

The participants of the marathon were all members of the *(website's name)* community. They received an invitation by e-mail to take part in the marathon. Most of them were from *(province name), (country name)*. We also had few participants from *(province name) and (country name)*. We have no reliable data on students' age, but the most frequent *(website's name)* users are Grades 6-8 (ages 12-14) which is a good approximation.

According to the context and problem of our study, as well as the theoretical framework, we investigated the following research questions:

1) What are participation patterns in the VMM and how are they tied to the task commitment of mathematically promising students?

2) What kinds of tasks were particularly difficult for the most persistent participants in our marathon?

ANALYSIS OF DATA ON STUDENTS' PARTICIPATION AND PROBLEM-SOLVING PATTERNS

Results on students' participation

The following table summarizes data for the overall participation and success rate.

Year	Number of participants	Number of solutions	Number of correct solutions	Correct solutions (in %)
2008	194	1379	703	50.9
2009	106	866	410	47.3
Total for two years	300	2245	1112	49.5

TABLE 1 Number of participants per year.

The number of participants was higher in the first year of the marathon (194 against 106). This can be explained, among others, by the fact that in 2008, we started the competition 2 weeks earlier before the end of classes. The success rate varies around 50% for each year, which can be considered as a good level of challenge. The following diagram represents participation trends through the whole marathon:

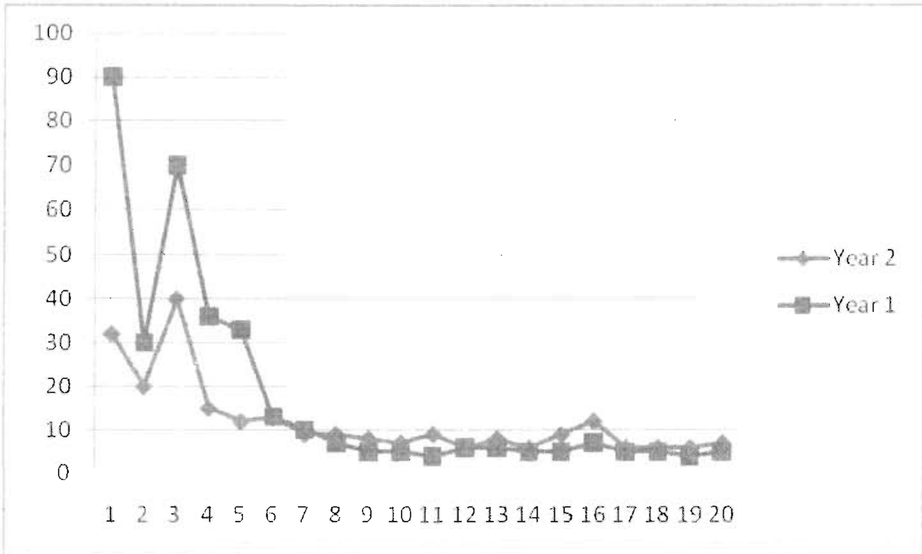

FIGURE 1 Participants' behaviour in VMM over two years (x-axes: set number; y-axes: number of participants).

Both broken lines represent almost the same pattern, which shows that the participation droped significantly after the first set, increased in the third, then droped again in the fourth, to slightly diminish in the fifth. After set 6, few participants remained in competition. However, this small group (having almost the same number of students from year to year and from set to set) preseveres solving problems until the end. We call these students as *persistent* particiants. Generally, persistent participants join the competitions in the first two sets and pursue the competion on a regular basis. But there are some participants that join the competions in later sets (for example, in the 16th set when the number slightly increased). Some participants left the marathon at some point and then came back.

We also notice that the number of solutions in even sets (2, 4, 6, etc.) is higher than in odd sets (1, 3, 5, etc.). In fact, the odd set started on Mondays and the even sets on Thursday. Perhaps students were looking for new problems on Monday like they were used to doing in regular problem-of-the-week activities.

Our data show that while the majority of students participated only a few times in the competition (1 to 5), there is small group of 11 participants (see 2 highlighted columns in Table 2) that persisted over a longer period of time; we call them the *most persistent* participants.

Number of students that participated in:	1-5 sets	6-10 sets	11-15 sets	16-20 sets	Total
VMM (2008)	184	4	3	3	194
VMM (2009)	95	6	3	2	106
Total	279	10	6	5	300

TABLE 2 Frequency of participation in the marathon.

The particularity of our marathon, when compared to other forms of mathematical competitions, is that no student selection is done over the competition. This way, pursuing the marathon depended exclusively on the personal choice of each participant.

Several reasons can affect the decision to continue or to withdraw from the competition. Some students may just having tried it once and then decided not to continue. Others may not have had access to a computer during the summer vacation. We have no reliable data regarding this. However, there may be a relationship between success at certain stages and the decision to continue participating. We defined as 'good days', the sets when students were able to solve all of the problems, and 'bad days', when they were not able to solve any problem correctly.

We learn from this data (see Table 3) that the success or failure in solving problems from one set may affect further participation in the VMM. In 2008, 55 students had a "bad day", following which 45 students (81.8 %) withdrew, 5 students (9.1%) took a break for a number of sets, and 5 students (9.1%) continued participating in the VMM. In 2009, 41 students had "bad days", following which 25 students (61.0%) dropped out, 2 students (4.9%) took a break for a number sets, and 14 students (34.2%) continued participating.

	Good days / student persistence N=34			Bad days / student persistence N=55		
	Students continued	Students had a break	Students dropped out	Students continued	Students had a break	Students dropped out
VMM 2008	19	2	13	5	5	45
			12 of 13 dropped out in the first 2 weeks			31 of 45 dropped out in the first 2 weeks
VMM 2009	N=21			N=41		
	15	3	3	14	2	25
			1 of 3 dropped out in the first 2 weeks			15 of 25 dropped out in the first 2 weeks

TABLE 3 Participation after "bad days" and "good days".

With students having a "good day", we discover an opposite pattern. Of the 34 students having "good days" in 2008, 13 (38.24%) dropped out immediately,2 (5.88%) had break, and 19 (55.88%) continued participating. Of the 21 students having "good days" in 2009, 3 (14.29%) dropped out immediately, 3 (14.29%) took a break, and 15 (71.43%) continued participating.

Furthermore, we learn that most of the students who dropped out after "bad days" and "good days" did it within the first two weeks of the VMM: 31/45 (68.89%) dropped out after a "bad day" and 12/13 (92.3%) after a "good day". One plausible explanation for this could be that the problems were either too difficult or too easy.

Additionally, the χ^2 *test* shows that decision making by students (for both year 1 and 2) conducted on variables "drop out", "having break" or "continue participating" in the VMM seems to be influenced by their result from the previous round. This means that students' behavior in the VMM was different after having had a "bad day" and a "good day":

(1) First year: $\chi^2(df = 2) = 23.459$ $***p < 0.001$ and (2) Second year: $\chi^2(df = 2) = 12.3554$ $***p < 0.001$.

Many factors can influence students' decision during the marathon including access to Internet or other similar factors not related to the activity's content or structure. It would be interesting to explore, if making "easy problem" easier and "hard problem" harder could motivate more students (at each end of the scale).

Results on students' problem-solving patterns

In order to find patterns in participants' problem-solving behaviour, we first analysed the number of solutions per category. Then we looked at the type of problems related to particular domains of the school curriculum. Finally, we analysed the solutions of 11 students that we considered as promising and persistent in order to look for types of problems that were the most challenging for them.

The following table presents the number of solutions submitted for each category of problem (1st – easiest level – 4th hardest level) as well as the number of correct solutions:

Number correct solutions of:	Level 1	Level 2	Level 3	Level 4
VMM (2008)	222 (344) – 64.5%	186 (321) – 57.9%	147 (317) – 46.4%	110 (318) – 34.5%
VMM (2009)	131 (225) – 50.2%	101 (204) – 49.5%	82 (206) – 39.8%	84 (200) – 42%
Total	353 (569) – 62%	287 (525) – 54.7%	229 (523) – 43.8%	194 (518) – 37.5%

TABLE 4 Number of solutions per level.

Data show that the number of students who tried to solve the first problem in the easiest set was almost always the same as the number of students who tried to solve more difficult problems. In year 1, the distribution of students per problem was: 344-321-317-318 and year 2: 225-204-206-200. The number of solutions for the first and easiest problem was slightly bigger (+7-10%) than the number of solutions from other categories. By comparing the percentage of correct solutions between the categories, we find that the number of correct solutions decreases according to the level both years included: 62% - 54.7% - 43.8% - 37.5%. We can conclude that the difficulty level attributed to each problem by our team of experts was generally correct.

According to the topics in New Brunswick's mathematics curriculum, problems can be divided into 7 groups: finding a pattern, algebra, probabilities, geometry, percentages, arithmetic, and number (common) sense. Some problems could be included in more than one category, so we decided to include them into the closest group (see Table 5).

	Patterns	Algebra	**Probabilities**	Geometry	**Percentages**	Arithmetic	Common (Number) sense
VMM (2008)	11	6	5	15	4	12	27
VMM (2009)	9	12	7	16	5	18	13
Total	20	18	12	31	9	30	40

TABLE 5 Distribution of problems according to the topics.

While we observe that the number of problem is unequal from one category to another, it reflects the unequal distribution of the topics in the school curriculum. For the group of 11 persistent and promising students identified in the previous section, we analyzed a number of solutions from each type of problems as well as number of correct solutions (see Table 6):

	Pattern	Algebra	**Probabilities**	Geometry	**Percentages**	Arithmetic	Common (number) sense
VMM (2008)	28 of 49 – 57.1%	14 of 29 - 48.3%	8 of 21 – **38.1%**	44 of 70 - 62.9%	16 of 23 - 69.6%	26 of 55 – 47.3%	85 of 131– 64.9%
VMM (2009)	21 of 31 - 67.8%	24 of 40 – 60%	10 of 29 - **34.5%**	29 of 64 – 45.3%	7 of 23 - **30.4%**	40 of 66 – 60.6%	34 of 52 – 65.4%
Total	61.2%	55.1%	**36.7%**	54.5%	50%	54.5%	65%

TABLE 6 Number of correct solutions per problem type.

The results for 11 promising students seem to show that problems with percentages (year 2) and probability (both years) were the most challenging. We briefly analyze two examples of such problems.

Probabilities

Two people are playing a game. Each of them is thinking of a two digit number. What are the odds that both people are thinking about the same number?

a) $\frac{1}{80}$

b) $\frac{1}{90}$

c) $\frac{1}{100}$

d) $\frac{1}{8100}$

e) $\frac{2}{8100}$

This problem was given in year 1. Four (4) of the six (6) promising and persistent students who attempted to solve it gave an incorrect answer. The answers were split between (d) 1/8100 (2 students) and (e) 2/8100 (2 students). The correct answer was (b). The reason for not choosing it may be explained as follows: *the first player can choose every two digit number and the probability of the same number being selected by the second player is* $\frac{1}{90}$. *The solution can be obtained by choosing every one of 90 two-digit numbers and then choosing the same number by the second player:*

$$\underbrace{\frac{1}{90}\times\frac{1}{90}+\frac{1}{90}\times\frac{1}{90}+\ldots+\frac{1}{90}\times\frac{1}{90}}_{90}=90\times\frac{1}{8100}=\frac{1}{90}$$

A possible reasoning leading to the first incorrect answer may be: $\frac{1}{90}\times\frac{1}{90}=\frac{1}{8100}$ and for the second: $\frac{1}{90}\times\frac{1}{90}+\frac{1}{90}\times\frac{1}{90}=\frac{2}{8100}$. Students were probably misled by the idea of probability of one of them (1/90) and the second (1/90) and just multiplied both numbers focusing on the number of players (2) instead of number of two digit numbers (90). The absence of explanation due to the multiple-choice form of questions keeps us from investigating students' thought process. Nevertheless, one plausible explanation can be the overall difficulties and misconceptions relative to probabilities, which are well documented (Garfield & Ahlgren, 1988; Jun & Pereira-Mendoza, 2002), and the particular type of non routine problems that are unusual for the school curriculum, although no special knowledge is required (Grugnetti & Jaquet, 2005).

Percentages

The success rate on problems with percentages given in the second year was significantly lower than that of the first year. This led us to investigate one example taken from the second year.

The circle is in the square. By what percentage is the area of the circle smaller than the area of the square? (a picture of the shapes was supplied)

a) 21,5%
b) 23%
c) 25%
d) 27,5%
e) Impossible to obtain

This problem was successfully solved by only one of the five students who attempted it. Three students chose the answer e) – impossible to obtain. While the problem appears to be a very "school-like" and can be solved by directly applying formulas learned in class. However, it may contain a hidden difficulty since no numbers were given with the picture.

The nature of problems on percentages forces the problem solver to focus on relationships that should be extracted from the text of the problem. For example, by setting the diameter of the circle equal to 1 unit (that means that the side of the square is also 1 unit), areas are respectively $\pi \times d^2/4$, so $\pi/4$ (circle) and the other area is $d^2=1$. Therefore, with $\pi=3.14$ we obtain 21.5% as the correct answer. One student who chose 23% may have used $\pi = 3.1$, which was the closest choice. Again, as in the previous example, the result seems to show insufficient attention to this kind of problem during regular in-class mathematics. It is our opinion that this finding should be investigated more closely.

FINDINGDS AND CONCLUSIONS

In this paper, we looked at participation patterns in the VMM and how they could be linked to the task commitment and problem solving abilities of mathematically promising students. This investigation focused on persistence and difficulties with particular types of problems.

Our first finding shows that there were more participants in the sets at the beginning of the week and less in those in the middle. We conclude that one set per week would probably be enough to keep students working at solving problems more consistently.

Our second finding is related to participation after having a "bad day" on one set compared to a "good day". Our data show that having a bad or good day may influence the decision to pursue the marathon or to withdraw, although there were significantly more students continuing after a "good day". We conclude that for the first 6 sets it would be better to chose 4 problems that are more 'spread out' in terms of difficulty. This would make easy problems even easier (to attract students who were not successful) and hard problems even harder (to keep those who may have left if they had not been challenged enough). Thus, we would better respond to criteria set up by Leikin (2009).

Third, we uncovered a very interesting fact: despite the different levels of difficulty in the 4 problems, students were trying to solve all of the problems once they started the set. Could this be explained by the fact that some points were also awarded for incorrect answers? While from the point of view of performance it is questionable to give points for incorrect answers, obtaining these points may actually motivate students to solve more problems and eventually develop better abilities. This is consistent with the Renzulli's model of mathematical giftedness (Renzulli, 1978).

The fourth finding is a success rate that corresponds, in general, to the scale of the four difficulty levels. This means that our problems were selected appropriately to gradually increase the level of challenge.

Fifth, by investigating the relations between mathematical challenge and students' abilities in terms of Krutetskii's characteristics (Krutetskii, 1976) we decided to focus on a small group of students who participated in more than half of the marathon. According to the literature, we can characterize these students as promising and persistent (Leikin, 2009). The multiple-choice structure makes relating their results to some ability levels impossible. Nonetheless, by looking at the tasks that were less successfully solved by these students, we found that two topics were more difficult for them: probabilities and percentages. A weaker performance on these kinds of tasks could be explained by the overall difficulty students generally have with both topics even in a regular classroom (Garfield & Ahlgren, 1988; Li & Pereira-Mendoza, 2002; White & Mitchelmore, 2008). However, it could also be related to the different (more complex) nature of mathematical relationships that need to be constructed in order to tackle these challenges with more success.

Classroom teachers could accomplish important work by offering enrichment or extracurricular activities, including online problem solving. Developing and studying such opportunities can be an interesting research task. In addition, from an organizational perspective, more needs to be done to increase students' participation, which could lead to some changes in the structure of the competition, in virtual tools, and in mathematical content. Different research tools like surveys, interviews, and more advanced ways of submitting solutions that would include students' explanations of their reasoning, would be helpful to get an insight into the impact of the VMM and to improve it.

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